Sensitivity of the Risk-of-Ruin

For Big Mike’s Trading Forum

Base Calculations

Substitute equations to find Rist of Ruin (RR) as function of propability of win (pWin), average win (xWin) and average loss (xLose).

In[1]:=
```
sol1 = FullSimplify[((1 - p)/p)^(R/A) /. p → 1/2 (1 + Z/A) /. Z → pWin xWin - (1 - pWin) xLose /.
    A → Sqrt[(pWin xWin^2) + ((1 - pWin) xLose^2)]]
```

Out[1]=
$$\left(\frac{xLose - pWin\,xLose - pWin\,xWin + \sqrt{xLose^2 - pWin\,xLose^2 + pWin\,xWin^2}}{(-1 + pWin)\,xLose + pWin\,xWin + \sqrt{xLose^2 - pWin\,xLose^2 + pWin\,xWin^2}}\right)^{\frac{R}{\sqrt{-(-1+pWin)\,xLose^2 + pWin\,xWin^2}}}$$

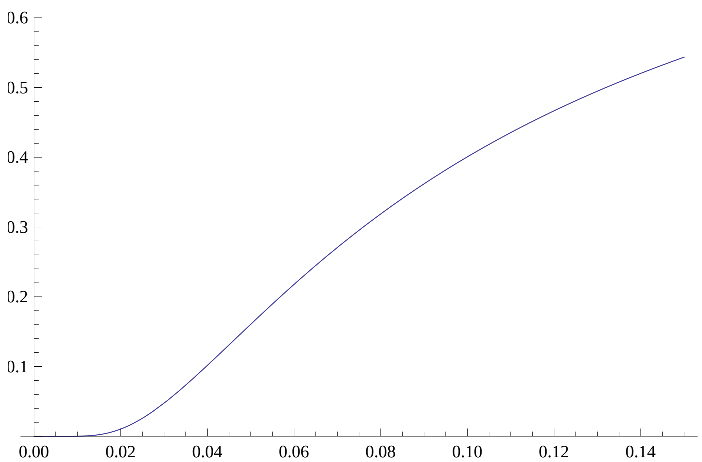
Next, we will perform a sleight of hand. Substitute xWin = f xLose, where f is the a scale factor between xWin and xLose (so, f = 2 means average wins are twice as big as average losers). This substitution scales xWin by f. This is important. Below, we are going to find the sensitivity of RR to these input variables. Causing RR to be sensitive to both xLose and xWin simultaneously is unrealistic. [Note that by not making this substitution, the calculations below become much more pessimistic].

In[2]:=
```
sol2 = FullSimplify[sol1 /. {xWin → f xLose}]
```

Out[2]=
$$\left(\frac{xLose - (1 + f)\,pWin\,xLose + \sqrt{\left(1 + \left(-1 + f^2\right) pWin\right) xLose^2}}{(-1 + pWin + f\,pWin)\,xLose + \sqrt{\left(1 + \left(-1 + f^2\right) pWin\right) xLose^2}}\right)^{\frac{R}{\sqrt{\left(1 + \left(-1+f^2\right) pWin\right) xLose^2}}}$$

Sanity check. Out plot below matches the source PDF.

In[3]:=
```
Plot[Evaluate[sol2 /. {R → 0.50, pWin → 0.4, f → 2}], {xLose, 0.0, 0.15},
 PlotRange → {0, 0.6}]
```

Out[3]=


Calculation

Let J be a vector containing the derivatives of LL with respect to pWin, f, and xLose.

```
J = {D[sol2, pWin], D[sol2, f], D[sol2, xLose]};
```

Next, calculate the sensitivity of LL to variations in the input variables. The sensitivity is the sum of the sensitivities due to each input. Each input sensitivity is the standard deviation/uncertainty of that input variable, scaled by the absolute value of the derivative of LL with respect to the variable. Thus, the sensitivity Δ is:

```
Δsum = Abs[J[[1]]] σpWin + Abs[J[[2]]] σf + Abs[J[[3]]] σxLose;
Δvec = {Abs[J[[1]]] σpWin, Abs[J[[2]]] σf, Abs[J[[3]]] σxLose};
Δmag = Sqrt[Δvec[[1]]^2 + Δvec[[2]]^2 + Δvec[[3]]^2];
```

There are conflicting opinions on how sensitivity should be calculated. Should it be the sum, or should each input variable sensitivity be considered sensitivity along that degree of freedom? If the latter, then the full sensitivity is the magnitude of this sensitivity vector. I will use this “vector calculation” below.

As another sanity check, for pWin of 0.4, f of 2, 5% xLose and 5% standard deviation for xLose, pWin and f, we calculate the mean RR, and RR worst-case:

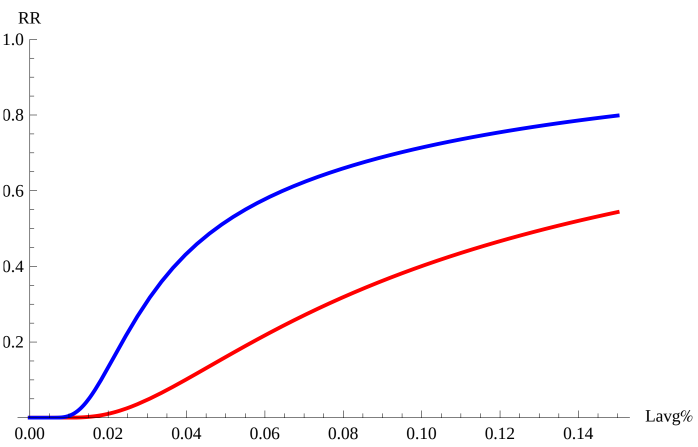
```
In[15]:= {sol2, sol2 + Δmag} /. {R → 0.50, pWin → 0.4, f → 2} /.
          {xLose → 0.05, σpWin → 0.05, σf → 0.05, σxLose → 0.05}

Out[15]= {0.160522, 0.517899}
```

Big difference. Plotting it out for 5% standard deviations:

```
In[24]:= Plot[
          Evaluate[{sol2, sol2 + Δmag} /.
            {R → 0.50, pWin → 0.4, f → 2, σpWin → 0.05, σf → 0.05, σxLose → 0.05}],
          {xLose, 0.00, 0.15}, PlotRange → {0, 1.0}, PlotStyle → {{Red, Thick}, {Blue, Thick}},
          AxesLabel → {"Lavg%", "RR"}]
```

Out[24]=



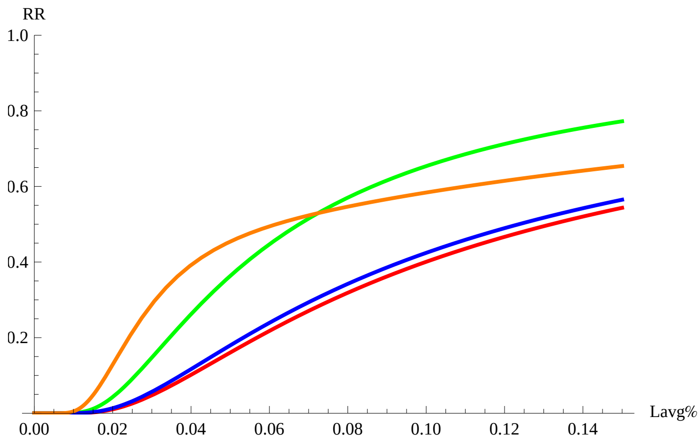
Decomposition

Which component is LL most sensitive to? We will set two of the three input variables to zero, and plot the 5% sigmas for each, individually:

```
In[25]:= sol3 = sol2 + Δmag /. {σpWin → σ, σf → 0, σxLose → 0};
         sol4 = sol2 + Δmag /. {σpWin → 0, σf → σ, σxLose → 0};
         sol5 = sol2 + Δmag /. {σpWin → 0, σf → 0, σxLose → σ};
```

In[31]:=
```
Plot[Evaluate[{sol2, sol3, sol4, sol5} /. {R → 0.50, pWin → 0.4, f → 2, σ → 0.05}],
 {xLose, 0.00, 0.15}, PlotRange → {0, 1.0},
 PlotStyle → {{Red, Thick}, {Green, Thick}, {Blue, Thick}, {Orange, Thick}},
 AxesLabel → {"Lavg%", "RR"}]
```

Out[31]=



In the above figure, the red line is the mean RR. Green includes only pWin sensitivity, blue only profit factor sensitivity, and orange sensitivity of xLose. From this we can conclude that in thsi case (pWin = 0.4, f = 2, and all standard deviations = 5%), RR is more sensivity to the size of xLose for conservative trades, and pWin for reckless trades.

Raw Equations

Simplifying the derivatives yields:

In[34]:=
```
FullSimplify[J[[1]]]
```

Out[34]=

$$-\left(R\,\mathrm{xLose}\left(\frac{\mathrm{xLose}-(1+f)\,\mathrm{pWin}\,\mathrm{xLose}+\sqrt{\left(1+\left(-1+f^2\right)\mathrm{pWin}\right)\mathrm{xLose}^2}}{(-1+\mathrm{pWin}+f\,\mathrm{pWin})\,\mathrm{xLose}+\sqrt{\left(1+\left(-1+f^2\right)\mathrm{pWin}\right)\mathrm{xLose}^2}}\right)^{\frac{R}{\sqrt{\left(1+\left(-1+f^2\right)\mathrm{pWin}\right)\mathrm{xLose}^2}}}\left(-2\,(1+(-1+f)\,\mathrm{pWin})\sqrt{\left(1+\left(-1+f^2\right)\mathrm{pWin}\right)\mathrm{xLose}^2}+\left(-1+f^2\right)(-1+\mathrm{pWin})\,\mathrm{pWin}\,\mathrm{xLose}\,\mathrm{Log}\left[\left(\mathrm{xLose}-(1+f)\,\mathrm{pWin}\,\mathrm{xLose}+\sqrt{\left(1+\left(-1+f^2\right)\mathrm{pWin}\right)\mathrm{xLose}^2}\right)\Big/\left((-1+\mathrm{pWin}+f\,\mathrm{pWin})\,\mathrm{xLose}+\sqrt{\left(1+\left(-1+f^2\right)\mathrm{pWin}\right)\mathrm{xLose}^2}\right)\right]\right)\right)\Big/\left(2\,(-1+\mathrm{pWin})\,\mathrm{pWin}\left(\left(1+\left(-1+f^2\right)\mathrm{pWin}\right)\mathrm{xLose}^2\right)^{3/2}\right)$$

In[35]:= `FullSimplify[J[[2]]]`

Out[35]=

$$-\left(R\,\text{xLose}\left(\frac{\text{xLose}-(1+f)\,\text{pWin}\,\text{xLose}+\sqrt{\left(1+\left(-1+f^2\right)\text{pWin}\right)\text{xLose}^2}}{(-1+\text{pWin}+f\,\text{pWin})\,\text{xLose}+\sqrt{\left(1+\left(-1+f^2\right)\text{pWin}\right)\text{xLose}^2}}\right)^{\frac{R}{\sqrt{\left(1+\left(-1+f^2\right)\text{pWin}\right)\text{xLose}^2}}}\left(2\sqrt{\left(1+\left(-1+f^2\right)\text{pWin}\right)\text{xLose}^2}+f\,(1+f)\,\text{pWin}\,\text{xLose}\,\text{Log}\left[\left(\text{xLose}-(1+f)\,\text{pWin}\,\text{xLose}+\sqrt{\left(1+\left(-1+f^2\right)\text{pWin}\right)\text{xLose}^2}\right)\Big/\left((-1+\text{pWin}+f\,\text{pWin})\,\text{xLose}+\sqrt{\left(1+\left(-1+f^2\right)\text{pWin}\right)\text{xLose}^2}\right)\right]\right)\right)\Big/\left((1+f)\left(\left(1+\left(-1+f^2\right)\text{pWin}\right)\text{xLose}^2\right)^{3/2}\right)$$

In[36]:= `FullSimplify[J[[3]]]`

Out[36]=

$$-\left(R\left(\frac{\text{xLose}-(1+f)\,\text{pWin}\,\text{xLose}+\sqrt{\left(1+\left(-1+f^2\right)\text{pWin}\right)\text{xLose}^2}}{(-1+\text{pWin}+f\,\text{pWin})\,\text{xLose}+\sqrt{\left(1+\left(-1+f^2\right)\text{pWin}\right)\text{xLose}^2}}\right)^{\frac{R}{\sqrt{\left(1+\left(-1+f^2\right)\text{pWin}\right)\text{xLose}^2}}}\text{Log}\left[\frac{\text{xLose}-(1+f)\,\text{pWin}\,\text{xLose}+\sqrt{\left(1+\left(-1+f^2\right)\text{pWin}\right)\text{xLose}^2}}{(-1+\text{pWin}+f\,\text{pWin})\,\text{xLose}+\sqrt{\left(1+\left(-1+f^2\right)\text{pWin}\right)\text{xLose}^2}}\right]\right)\Big/\left(\text{xLose}\sqrt{\left(1+\left(-1+f^2\right)\text{pWin}\right)\text{xLose}^2}\right)$$

From the equations above, you can calculate teh sensitivity via:

$$\Delta = J_1\,\sigma_{\text{pWin}}+J_2\,\sigma_f+J_3\,\sigma_{\text{xLose}}$$

or from:

$$\Delta = \sqrt{\left(J_1\,\sigma_{\text{pWin}}\right)^2+\left(J_2\,\sigma_f\right)^2+\left(J_3\,\sigma_{\text{xLose}}\right)^2}$$