Non-Periodic Australian Stock Market Cycles:
Evidence from Rescaled Range Analysis

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Abstract

The standard compliment of statistical techniques used to identify predictable market structure assume that the data are independent and identically distributed. Further, they are only capable of identifying regular periodic cycles. Yet, financial returns data are not independent and cycles are most probably not periodic. Rescaled range analysis is a nonparametric technique which is able to distinguish the average cycle length of irregular cycles. Using Australian stock market data, this paper finds evidence of long memory in the returns generating process and non-periodic cycles of approximately 3, 6 and 12 years in average duration.

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1 INTRODUCTION

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Traditionally, events are viewed as either random or deterministic. For markets, the failure of standard statistical analysis to uncover any long run trends or cycles has led most to the conclusion that markets must be the former rather than the latter. However, market returns are not normally distributed as they have been found to be leptokurtic insomuch as the data exhibits a disproportionately greater number of very large and very small changes relative to that of a normal distribution. Further, volatility in the market does not scale according to the square root of time.[1] Both of these observations suggest the possible presence of a long memory system generated by a nonlinear stochastic process. The standard compliment of statistical techniques are not well suited to identifying any nonlinear structure in market data. Further, the leptokurtosis common to financial time series data violates the assumption that the data are independent and identically distributed (iid).[2]

Nonparametric statistical techniques provide a viable alternative to testing for such nonlinear structure and are ideal for modelling financial data as they make no prior assumption about the distribution of the data. One such nonparametric technique is rescaled range analysis first introduced by Hurst (1951) and subsequently refined *inter alia* by Mandelbrot (1972, 1982), Mandelbrot and Wallis (1969) and Lo (1991).[3] Rescaled range analysis centers on the proposition that the dispersion of the returns generated by a truly random process will scale according to $H = 0.5$ in $D = c * n^H$ where D is the dispersion of returns, c is a constant and n is a measure of time.[4] However, if the dispersion of returns scales at a rate faster than that of a random walk, ($H > 0.50$), the return generating process must be related in some way. On the other hand, where a series reverses itself more often than a random walk, it is antipersistent

[1] An assumption necessary to apply the normal distribution is that the standard deviation of a series at one frequency will scale to that of another frequency by multiplying it by the square root of time. For example, we may annualise monthly standard deviation estimates by multiplying them by $\sqrt{12}$.

[2] Technically speaking, the use of the standard compliment of statistical tools requires independence of the data or, at best, a very short memory process. Where data is not iid, one may manipulate the data to create an ‘approximately normal’ distribution by removing outliers and renormalising the data. This process allows the standard tools to be applied with some modifications. Whilst this process can be justified in some instances, in the context of financial data modelling the case is not clear.

[3] For a survey see Brock and de Lima (1996) pp. 339 - 341.

[4] This equation is a generalised form of Einstein’s (1908) $R = n^{0.50}$ formula for estimating the distance travelled by a particle in Brownian motion which is the primary model for a random walk process.

(H < 0.5). Thus, rescaled range analysis is a robust non-parametric technique for testing whether or not a market is truly random.[5] An added advantage of rescaled range analysis lay in its ability to discern cycles within data. This not only includes regular periodic cycles but non-periodic cycles as well. These non-periodic cycles may be either a biased random walk in which the bias changes at indeterminate intervals or the result of a nonlinear dynamic system.

The purpose of this paper is to apply rescaled range analysis to Australian stock market data in an attempt to establish the presence of long memory and market cycles. One problem with R/S analysis is that is it data intensive, requiring data to be drawn over a long sample period. A common analogy used to highlight this problem comes from the field of meteorology. It is well established that the sun exhibits an 11 year sunspot cycle and to correctly identify this phenomena would require many observations taken over a long period of time. In the absence of data sampled over a sufficiently long enough period, taking more observations over a short sample period would not suffice. For example, testing five years of high frequency data would not reveal this sunspot pattern regardless of the number of observations taken over the sample period. This problem is most common for financial market research as most asset price time series have only been collected for a relatively short period of time. In the current context, a number of papers have applied R/S analysis to stock market returns and in each case failed to reject the null hypothesis of a random walk (see *inter alia* Lo (1991) and Huang and Yang (1995)).[6] However, the sample period for the data considered in each of these papers was limited. For example, Huang and Yang (1995) only sampled data over the period January 1988 to June 1992. An exception may be found in Peters (1994) who applied R/S analysis to the Dow Jones industrial index sampled over the period 1888 to 1991 and found evidence of a two month and four year non-periodic cycle. An even longer time series exists for the Australian stock market which consists of monthly returns to an equally weighted national stock market index sampled over the period 1875 to 1996. This index is a composite derived from a number of sources and was constructed with the intention of being comparable to the

[5] An alternative to R/S analysis is the Variance Ratio test which assumes normality in the data.

returns to the All Ordinaries Index. From 1875 to 1935 the index is the Commercial and Industrial Index; from 1936 to 1957 the index is the Lamberton Market Index[7]; from 1958 to 1973 the index is the AGSM Equally Weighted Industrial Index; and from 1974 to 1996 the index is the AGSM Equally Weighted Market Index.[8] To the extent that national stock markets are heterogenous, the study of markets outside the US is interesting as we may find a different pattern of cycles to those already identified in previous research.[9] The 121 year history of the returns to the Australian stock market provides an ideal sample for the application of R/S analysis. The remainder of this paper proceeds as follows. Section 2 introduces rescaled range analysis and discusses its application to financial time series data. Section 3 details the data to be used in this study and presents the results of the analysis. Section 4 considers the market cycles uncovered by the rescaled range analysis. Finally, Section 5 presents some conclusions.

2 EMPIRICAL FRAMEWORK

As a first step to rescaled range analysis, it is necessary to prewhiten the data using an Auto-Regressive (AR) model as the presence of linear dependence in the data can favourably bias the results ie., increase the likelihood of detecting the presence of a long-memory process. While an AR model does not eliminate all linear dependence, Brock, Dechert, Sheinkman and Le Baron (1996) propose that the effect of any linear dependence in the residuals will be insignificant.[10] Thus, rescaled range analysis is applied to the residuals (ε_t) of an AR model. As a first step, the sample (N) was split into A contiguous subperiods of length n (where n is an integer which evenly divides

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[6] R/S analysis has been applied to a wide variety of asset prices and economic aggregates. For evidence on gold prices refer to Cheung and Lai (1993); futures see Corazza, Malliaris and Nardelli (1997); and exchange rates see Batten and Ellis (1996).

[7] Details of the Lamberton Market Index may be found in Brailsford and Easton (1991).

[8] The author would like to thank Tim Brailsford for making the data available.

[9] See Brailsford and Faff (1993) for a discussion of the characteristics which distinguish the Australian stock market from the US market.

[10] An alternative technique for dealing with short term dependence has been proposed by Lo (1991). Whilst this technique has the advantage of allowing formal hypothesis testing, it suffers from two shortcomings as it is sensitive to moment conditional failure (see Hiemstra and Jones (1994)) and the results are extremely sensitive to the choice of autocorrelations included in the Newey-West heteroscedasticity and autocorrelation consistent estimator.

into the sample length).[11] Each subperiod may be denoted as I_a (where $a = 1 \ldots A$) and each element of I_a may be denoted as $N_{k,a}$ (where $k = 1 \ldots n$). For each I_a, we must rescale the data by subtracting the subperiod mean:

$$Y_{k,a} = \left(N_{k,a} - e_a\right) \qquad k = 1 \ldots n \tag{1}$$

where e_a is the average value of the elements in subperiod I_a and may be estimated as:

$$e_a = \frac{\sum_{k=1}^{n} N_{k,a}}{n} \tag{2}$$

The resulting series ($Y_{k,a}$) has a mean of zero and if we cumulatively sum $Y_{k,a}$ we get the series ($X_{k,a}$) in which the last value will always be zero since the series by construction has a mean value of zero. The range of the cumulative series $X_{k,a}$ may be estimated as :

$$R_{I_a} = \max(X_{k,a}) - \min(X_{k,a}) \qquad \text{where } 1 \le k \le n \tag{3}$$

We may normalise R_{I_a} to create the rescaled range by dividing each range by the sample standard deviation (S_{I_a}), ie. $= R_{I_a} / S_{I_a}$, where S_{I_a} may be estimated as :

$$S_{I_a} = \frac{\sqrt{\sum_{k=1}^{n} (N_{k,a} - e_a)^2}}{n} \tag{4}$$

The use of rescaled ranges is important as it allows the direct comparison between periods spread across time. This is potentially useful in nominal financial time series analysis where inflation typically limits direct comparison. This process will result in A rescaled ranges and the average rescaled range across the whole sample for subperiod length n (R/S) may be estimated as :

$$(R/S)_n = \frac{\sum_{a=1}^{A} (R_{I_a} / S_{I_a})}{A} \tag{5}$$

[11] The use of contiguous subperiods means there is no overlap in the data (ie., 1 … n, n+1 … 2n, 2n +1 … and so on).

Typically, a number of integers will evenly divide into the sample length and so this process must be repeated for all successive values of *n*. Indeed, the sample length should be chosen so as to maximise the number of integers which evenly divide into it.[12] To test the significance of R/S it may be compared to the expected R/S value (E(R/S)) which is implied by a 'true' random walk process and may be derived as :

$$E(R/S)_n = \left(\frac{n-0.5}{n}\right) * \sqrt{\frac{2}{np}} * \sum_{r=1}^{n-1} \sqrt{\frac{n-r}{r}} \qquad (6)$$

This equation is a modified version of that provided by Anis and Lloyd (1976) with an error correction factor to account for its small sample bias (Peters (1994) p. 69).

The dispersion of the returns generated by a truly random process will exhibit a Hurst coefficient of H value of 0.50 in D = c * n^H where D is the dispersion of returns, c is a constant and *n* is a measure of time.[13] The Hurst coefficient may be estimated using an OLS regression of the following form :

$$\log(R/S)_n = \log(c) + H * \log(n) + e_t \qquad (7)$$

Further details on this estimation procedure and the relationship between the R/S statistic and the Hurst exponent maybe found in Cutland, Kopp and Willinger (1993). H values greater than 0.50 imply the dispersion of returns scales at a rate faster than that of a random walk. As such, the return generating process must be related in some way and such persistence is characterised by long memory effects. H values of less than 0.50 imply the dispersion of returns scales at a slower rate than that of a random walk. Thus, the series is antipersistent in that it will cover less distance compared to a random series. Whilst not suggesting mean reversion (the system does not have a stable mean and so there is no mean to revert to), it does indicate that the system

[12] In R/S analysis, the sample length should be chosen so as to maximise the number of integers which evenly divide into the sample and so generate the greatest number of R/S values. For example, a data set of 499 observations has only 2 divisors. It is better for the sample length to be reduced to allow a greater number of divisors such as a sample of 450 data points which has 9 divisors.

[13] The H or Hurst coefficient was named by Mandelbrot in honour of H.E. Hurst, the creator of rescaled range analysis.

reverses itself more frequently than a random one. As the E(R/S) values are random normally distributed variables, the expected values of the Hurst coefficient (E(H)) are also normally distributed. In this case, the expected variance of the Hurst coefficient would be 1/T where T is the total number of observations in the sample. Thus, to test whether the generated *H* coefficient is significant its value should be approximately two standard deviations away from the E(*H*) value where the standard deviation is estimated as $\sigma = \sqrt{1/T}$.

Estimating and comparing the Hurst coefficient to its expected value is one way to determine the presence of long memory in the data. A second technique uses the V-statistic which may be estimated as :

$$\text{V-statistic}_n = (R/S)_n / \sqrt{n} \tag{8}$$

In V-statistic / log(*n*) space, V is theoretically a horizontal line if the R/S statistic was scaling with the square root of time ie. the data is random and independent. However, if the process by which the data is generated is persistent (antipersistent), then R/S scales at a faster (slower) rate than the square root of time and so would be positively (negatively) sloped (Peters (1994) p. 92). The use of the V-statistic has an additional advantage in that it is able to discern cycles within data. This not only includes regular periodic cycles but non-periodic chaotic cycles as well. The latter refers to deterministic non-linear dynamic systems which, whilst erratic in behaviour, possess an average cycle length.[14] For example, a stock market may exhibit a annual cycle. However, the erratic nature of this cycle means that its duration is only one year on average and any given cycle may actually last a longer or shorter period of time. The presence of such periodic and non-periodic cycles can be identified by a plot of the V-statistic. Where the slope of the V-plot flattens out, the long memory process has dissipated indicating the end of the cycle. If the V-plot subsequently resumes its

[14] A second type of periodic cycle is the statistical cycle which refers to a biased random walk price generating process. The bias in this process changes in response to periods of economic reversal which itself is unpredictable (such as coming out of a bear or a bull run). For such statistical cycles, there is no average cycle length as the arrival of the reversal is a random event. As such, the V-statistic is unable to identify such cycles and the V-statistic plot will not deviate from its trend.

upward path, a longer cycle exists in the data. Thus, the V-plot is able to identify multiple cycles where they exist.[15]

3 RESULTS

The data used in this study consists of monthly Australian stock market returns sampled over the period April, 1876 to March, 1996 giving a total of 1440 observations.[16] These returns are the log price relative of a composite representative national stock market index. Figure 1 presents the values of a national stock market index reconstructed using these returns and assuming a base value of 100 (log index values are presented as they provide a better diagramatic representation of the data). From Figure 1, the Australian stock market has generally appreciated over time and the crashes of 1930 and 1987 are clearly visible. The descriptive statistics of the data indicate a mean monthly return of 0.93% and a standard deviation of 4.03%. Most importantly, the return series exhibits skewness (S = 0.146), excess kurtosis (11.86) and fails the Jarque Bera test of normality (JB = 4761.0). The presence of leptokurtosis in the data provides evidence suggestive as to a systematic bias in the data generating process.

[FIGURE 1 ABOUT HERE]

To test the nature of this systematic bias, rescaled range analysis may be applied. As a first step it is necessary to prewhiten the data by estimating an AR equation of the following form :

$$R_t = \alpha_0 + \beta_1 R_{t-1} + \chi_1 D + \varepsilon_t \tag{9}$$

where R_t are the returns to the Australian share market, D is a dummy variable which takes on a value of unity for the month of the October 1987 Crash, α, β and χ, are

[15] There is a limit to the number of cycles which can be identified. As a general rule, where more than four cycles are present in the data, they tend to merge. At the limit of an infinite number of cycles, the log(R/S) plot would not deviate from trend which would normally be taken as indicative of no average cycle length.
[16] The data was available from February 1875 (1454 observations), but the sample was manipulated to a length which could be divided by the greatest number of integers after the estimation of the mean model.

coefficients to be estimated and ε_t are the residuals from the model.[17] Equation (9) was fitted to the data and the estimated results are :

$$R_t = 0.0073 + 0.2536\, R_{t-1} - 0.3952 \text{ Crash Dummy}$$
$$\quad (7.35) \quad (10.29) \qquad (10.71)$$

$$\bar{R}^2 = 0.1274 \qquad \text{S.E.} = 0.0368$$
$$\text{DW} = 2.0351 \qquad \text{F-Statistic} = 106.11$$

Note : t-statistics in parentheses

Rescaled range analysis may be applied to the residuals of this AR model. As a first step, it is necessary to divide the sample into subperiods of length *n*. For financial time series, Peters (1994) suggests $n \approx 10$ as a starting point as values of $n < 10$ have been found to produce unstable estimates when sample sizes are small (Peters (1994) p. 63). Thus, the data may be split into 144 contiguous subperiods of 10 observations. In each 10 observation subperiod, the data is rescaled to a mean value of zero by subtracting the sample mean (estimated as Equation (2)) from each observation. The cumulative sum of these ten rescaled values was then generated and the range determined according to Equation (3). Each range was then normalised and the average R/S_{10} value across all subperiods calculated. The process was then repeated for higher values of n which were evenly divisible by the total sample. Thus, the process was repeated for n = 12, 15, 16, 18, … . The estimated R/S values are presented in Table 1 as are the E(R/S) coefficients which were generated according to Equation (6). To aid in the interpretation of these estimates, Figure 2 presents a plot of log(R/S) and log(E(R/S)) against log(n). One can see that the R/S values scale closely to the E(R/S) values until n = 32 (log(n) = 1.50) as evidenced by the parallel paths of the two plots. After this point however, a systematic deviation of the R/S and E(R/S) estimates is visible until a break in this trend which appears at n = 144 (log(n) = 2.15)

[FIGURE 2 AND TABLE 1 ABOUT HERE]

[17] This dummy variable was only used to account for the 1987 crash as it was unique in that it occured in a very short period of time. Other crashes experienced by the Australian market were more prolonged.

To investigate this deviation of the R/S and E(R/S) series, Figure 3 presents a plot of the V-statistic against log(n). If the series is persistent, this ratio will increase and a plot of the V-statistic will exhibit a positive slope. For non-periodic statistical cycles in which there is no average cycle length, the plot of the V-statistic for R/S will not deviate from this upward trend. However, a deterministic system which possess an average cycle length will deviate from its trend line at the end of its cycle. Where more than one cycle is discernible in the data, a break in the positive slope of the log(R/S) plot will appear at the end of one cycle before the plot resumes its upward trend to again deviate at the end of the next cycle. From Figure 3, the plot of the V-statistic for the monthly Australian stock market data indicates persistence in the returns generating process as the ratio is increasing at a faster rate than that of the V-statistic estimated for the E(R/S) series. More specifically, the local slope for $n \leq 32$ is not easily distinguishable from a random walk. Where $n > 32$ however, the V-statistic for R/S diverges from the V-statistic for E(R/S) until $n > 144$ (approximately 12 years). For values of $n > 144$, the two series again converge with the exception of the observation at $n = 180$. Thus, from $n = 32$ to 144 the local slope in this region increases at a faster rate than that implied by a random walk. An additional cycle not immediately obvious in Figure 2, may be identified as ending at $n = 72$ (approximately 6 years). The break in the upward trend of the plot of the V-Statistic at $n = 72$ indicates the end of a non-periodic cycle. However, the resumption of the upward trend shortly thereafter is consistent with the presence of a longer cycle in the data. These results clearly support the presence of a non-linear dynamic system driving the data and suggest the presence of cycles in the stock market which exhibit chaotic behaviour.

[FIGURE 3 ABOUT HERE]

To assess the significance of these visual clues as to cycle length, Equation (7) was applied to both the R/S and E(R/S) values and H and E(H) coefficient estimated initially over the period $32 \geq n \geq 144$. The results are presented in Table 2. The estimated Hurst coefficient is 0.622 and for the E(R/S) series the E(H) coefficient is 0.574. As the H estimate is 1.85 standard deviations greater than the E(H) coefficient (standard deviation = 0.026), we may conclude that they are significantly different at the 10% level and so reject the null hypothesis that the system is an independent process. H may also be estimated within each subperiod identified by Figure 3, ie. H may be estimated over the period $32 \geq n \geq 72$ and $80 \geq n \geq 144$ and the results are again presented in Table 2. For the first subperiod, H (= 0.699) indicates that R/S increases at a faster rate than suggested by a random process (E(H) = 0.589) and the difference is statistically significant (H is 4.17 standard deviations greater than E(H)). Thus, the market clearly exhibits persistence not consistent with a random walk over this horizon. In the second subperiod, H (= 0.598) is 1.70 standard deviations greater than E(H) (= 0.554) and so is significant at the 10% level.

[TABLE 2 ABOUT HERE]

Overall, these results suggest that the returns generating process in the Australian stock market is persistent and a 6 year (possibly an average business cycle) and a 12 year non-periodic cycle exist in the Australian stock market. The identification of a six year average cycle in the data is broadly consistent with previous estimates of the business cycle. For example, Layton (1997a,b) studied the Australian economy over the period 1950 - 1994 using a coincident index and found evidence of a business cycle which has a median length of 5.25 years. Unfortunately, a lack of R/S estimates prevents a more precise identification of the length of this cycle.

3.1 DAILY DATA

In general, the results obtained using monthly data suggest the presence of two cycles within the Australian stock market which possess a 6 and 12 year duration on average. The cycles evident in the monthly data should be evident in more frequently sampled data if they are not a statistical anomaly. That is to say, daily returns should exhibit a non-periodic cycle of 6 years (ie, 1500 daily observations). Further, if the 12 year cycle is a genuine non-periodic cycle and not a stochastic boundary due to sample size, its presence should be largely independent of the sampling frequency.

The use of more frequently sampled data does however, bring with it some problems as it typically contains greater levels of noise compared to less frequently sampled data. Whilst R/S analysis is robust to the presence of noise (Peters (1994) p. 98 - 101), its presence may bias the Hurst exponent downward. Further, more frequently sampled data is typically only available for relatively short periods of time and so suffer from the 'sunspot' dilemma discussed in Section 1. This makes it difficult to uncover long cycles in the data such as the 12 year cycle found in Section 3. These concerns aside, this study will consider Australian stock market returns data sampled at a higher frequency. Daily returns to the Australian All Ordinaries stock market index were available for the period December, 1980 to August, 1998 giving a total of 4862 observations.[18] Equation 9 was fitted to this data and the results may be summarised as:

$$
\begin{array}{l}
R_t = \underset{(2.71)}{0.0003} + \underset{(8.60)}{0.1110}\, R_{t-1} - \underset{(31.53)}{0.2837}\ \text{Crash Dummy}
\end{array}
$$

$$
\begin{array}{ll}
R^2 = 0.1833 & \text{S.E.} = 0.0089 \\
\text{DW} = 1.9262 & \text{F-Statistic} = 1.9262
\end{array}
$$

Note : t-statistics in parentheses

Rescaled range analysis was applied to the residuals of this AR equation and the results are presented in Table 3. In contrast to the monthly data, none of the daily R/S estimates are less than the E(R/S) values. This can be seen in Figure 4 which plots the log(R/S) and log(E(R/S)) values against log(n). Up to n = 30 (log(n) = 1.47), the R/S and E(R/S) values scale closely and their difference varies little in one direction or the other. Where n > 30 however, a systematic deviation between the R/S and E(R/S) values is evident. This difference is most obvious for values of n > 243 (log(n) = 2.38) and the trend reverses after n = 1620. Further, the possibility of multiple cycles within the data exists as a number of break points in the data are apparent at n = 270 (approximately 1 year) and 810 (approximately 3 years). This is more obvious in Figure 5 which plots the V-statistic against log(n). Interestingly, the break point at n = 1620 roughly corresponds to the 6 year cycle identified using the monthly data (1620

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[18] Weekly Australian stock market returns sampled over the same period were also considered and the results (not reported) were broadly consistent with those reported for the daily data. They are available upon request.

days / 252 trading days per year $\approx$ 6.4 years). This would tend to suggest that the 6 year nonperiodic cycle found in the monthly data is a deterministic system which possesses the average cycle length identified. Unfortunately, a lack of observations prevents an analysis of the 12 year cycle identified in the monthly data.

[TABLE 3, FIGURES 4 AND 5 ABOUT HERE]

The Hurst coefficient may be estimated initially over the period $30 \geq n \geq 1620$ and the results are presented in Table 4. The estimated Hurst coefficient is 0.571 and for the E(R/S) series the E(H) coefficient is 0.542. As the H estimate is 2.027 standard deviations greater than the E(H) coefficient (standard deviation = 0.0143), we may reject the null hypothesis that the system is an independent process. H may also be estimated within each of the subperiods identified in Figure 5 and the results are again presented in Table 3. For the subperiod $30 \geq n \geq 270$, H (= 0.568) is not significantly greater than the E(H) estimate (= 0.563). Thus, it is difficult to distinguish the market from a random walk in the short term. However, in the subperiod $324 \geq n \geq 810$, H (= 0.702) is 12.6 standard deviations greater than E(H) (= 0.522) and so is significant at the 1% level. Although hampered by a lack of observations, in the final subperiod $972 > n > 1620$ the estimated H coefficient (= 0.763) is significantly different to the E(H) value of 0.513. Unfortunately, the data intensive nature of the rescaled range process means that validating the 12 year cycle found in the monthly returns is not possible using the daily returns.

[TABLE 4 ABOUT HERE]

4 CONCLUSIONS

In fractal analysis, chaos and order are seen to coexist in a relationship of local randomness and global determinism. Most natural systems may be seen to operate in this way and markets may also be locally random, but have a global statistical structure that is non-random. Rescaled range analysis is a robust nonparametric statistical technique which can discern the presence of fractal structure in financial time series data. In this paper, rescaled range analysis was applied to Australian stock market data and the market was found to exhibit a highly degree of persistence once short term effects were removed. Further, the stock market has nonlinear dynamic cycles of approximately 3, 6 and 12 years. This result has important implications for momentum and other forms of technical analysis as well as guiding the appropriate period length for model development. One limiting factor of the analysis was the lack of sufficient data to complete the full range of tests on more frequent returns to reinforce these monthly data based findings.

TABLE 1

Rescaled Range Analysis Results - Monthly Returns Data

The following table presents the R/S_n and $E(R/S)_n$ values for various sample lengths (n) estimated for monthly Australian stock market returns sampled over the period 1876 to 1996. The corresponding V-statisic estimated according to Equation (8) is presented in the final two columns.

Subperiod Sample Length (*n*)	$(R/S)_n$	$E(R/S_n)$	V - statistic $(R/S)_n$	V - statistic $E(R/S_n)$
10	2.8537	2.6503	0.9024	0.8381
12	3.2147	3.0374	0.9280	0.8768
15	3.7339	3.5605	0.9641	0.9193
16	3.9168	3.7228	0.9792	0.9307
18	4.2045	4.0323	0.9910	0.9504
20	4.3153	4.3247	0.9649	0.9670
24	5.0060	4.8677	1.0218	0.9936
30	5.6513	5.6018	1.0318	1.0227
32	5.7609	5.8295	1.0184	1.0305
36	6.4836	6.2642	1.0806	1.0440
40	7.0028	6.6749	1.1072	1.0554
45	7.4658	7.1600	1.1129	1.0673
48	7.9205	7.4380	1.1432	1.0736
60	9.1423	8.4704	1.1803	1.0935
72	10.3217	9.4026	1.2164	1.1081
80	10.7489	9.9810	1.2018	1.1159
90	11.3777	10.6643	1.1993	1.1241
96	11.8514	11.0560	1.2096	1.1284
120	13.3877	12.5111	1.2221	1.1421
144	15.3123	13.8258	1.2760	1.1522
160	15.5147	14.6417	1.2265	1.1575
180	17.4002	15.6058	1.2969	1.1632
240	18.8509	18.2127	1.2168	1.1756
288	19.9099	20.0691	1.1732	1.1826
360	22.0340	22.5831	1.1613	1.1902
480	21.8686	26.2660	0.9982	1.1989

TABLE 2
Hurst Coefficient Estimation - Monthly Data

The following table presents the estimated results for Equation (7) applied to the R/S and E(R/S) values presented in Table 1 over the subperiods identified in the first column.

n	R/S_n	$E(R/S_n)$
$32 \geq n \geq 144$	$\log (R/S)_n = -0.155 + 0.622 \log (n)$ (6.12) (44.51) $R^2 = 0.994$ S.E. = 0.009	$\log (R/S)_n = -0.095 + 0.574 \log (n)$ (17.94) (196.5) $R^2 = 0.999$ S.E. = 0.002
$32 \geq n \geq 72$	$\log (R/S)_n = -0.282 + 0.699 \log (n)$ (7.81) (32.28) $R^2 = 0.995$ S.E. = 0.006	$\log (R/S)_n = -0.119 + 0.589 \log (n)$ (23.14) (189.5) $R^2 = 0.999$ S.E. = 0.001
$80 \geq n \geq 144$	$\log (R/S)_n = -0.112 + 0.598 \log (n)$ (2.19) (23.57) $R^2 = 0.994$ S.E. = 0.005	$\log (R/S)_n = -0.055 + 0.554 \log (n)$ (18.32) (371.8) $R^2 = 0.999$ S.E. = 0.001

TABLE 3
Rescaled Range Analysis - Daily Data

The following table presents the R/S_n and $E(R/S)_n$ values for various sample lengths (n) estimated for daily Australian stock market returns sampled over the period 1980 to 1998. The corresponding V-statisic estimated according to Equation (8) is presented in the final two columns.

Subperiod Sample Length (*n*)	$(R/S)_n$	$E(R/S_n)$	V - statistic $(R/S)_n$	V - statistic $E(R/S_n)$
10	3.1268	2.6502	0.9888	0.8381
12	3.5217	3.0373	1.0167	0.8768
15	4.0070	3.5605	1.0346	0.9193
18	4.4749	4.0323	1.0548	0.9504
20	4.7872	4.3247	1.0705	0.9670
27	5.7880	5.2451	1.1139	1.0094
30	5.9919	5.6017	1.0940	1.0227
36	6.8470	6.2641	1.1412	1.0440
45	7.7896	7.1599	1.1612	1.0673
54	8.7513	7.9687	1.1909	1.0844
60	9.4840	8.4703	1.2244	1.0935
81	11.1388	10.0511	1.2376	1.1168
90	11.4992	10.6642	1.2121	1.1241
108	12.8044	11.8039	1.2321	1.1358
135	14.3617	13.3467	1.2361	1.1487
162	16.2045	14.7407	1.2731	1.1581
180	16.9675	15.6058	1.2647	1.1632
243	19.7950	18.3339	1.2699	1.1761
270	21.9420	19.3925	1.3354	1.1802
324	22.0440	21.3611	1.2247	1.1867
405	27.1934	24.0273	1.3513	1.1939
486	30.1824	26.4372	1.3691	1.1992
540	32.5311	27.9331	1.3999	1.2021
810	42.5392	34.4833	1.4947	1.2116
972	42.6738	37.8896	1.3688	1.2153
1215	50.4901	42.5038	1.4485	1.2194
1620	63.0240	49.2644	1.5658	1.2240
2430	67.5922	60.6044	1.3712	1.2294

TABLE 4
Hurst Coefficient Estimation - Daily Data

The following table presents the estimated results for Equation (7) applied to the R/S and E(R/S) values presented in Table 3 over the subperiods identified in the first column.

n	R/S_n	$E(R/S_n)$
$30 \geq n \geq 1620$	$\log (R/S)_n = -0.054 + 0.571 \log (n)$ (3.96) (97.15) $R^2 = 0.997$ S.E. = 0.013	$\log (R/S)_n = -0.037 + 0.542 \log (n)$ (5.21) (179.2) $R^2 = 0.999$ S.E. = 0.007
$30 \geq n \geq 270$	$\log (R/S)_n = -0.047 + 0.568 \log (n)$ (3.08) (73.09) $R^2 = 0.997$ S.E. = 0.008	$\log (R/S)_n = -0.076 + 0.563 \log (n)$ (10.87) (159.0) $R^2 = 0.999$ S.E. = 0.003
$324 \geq n \geq 810$	$\log (R/S)_n = -0.408 + 0.702 \log (n)$ (4.50) (20.87) $R^2 = 0.993$ S.E. = 0.010	$\log (R/S)_n = 0.018 + 0.522 \log (n)$ (6.13) (475.2) $R^2 = 0.999$ S.E. = 0.001
$972 \geq n \geq 1620$	$\log (R/S)_n = -0.651 + 0.763 \log (n)$ (43.74) (158.6) $R^2 = 0.999$ S.E. = 0.001	$\log (R/S)_n = 0.043 + 0.513 \log (n)$ (26.0) (959.4) $R^2 = 0.999$ S.E. = 0.001

FIGURE 1 - Australian Stock Market Index 1875 - 1996

This diagram plots the log of the Australian stock market index reconstructed from returns data using a base value of 100 for 1875.

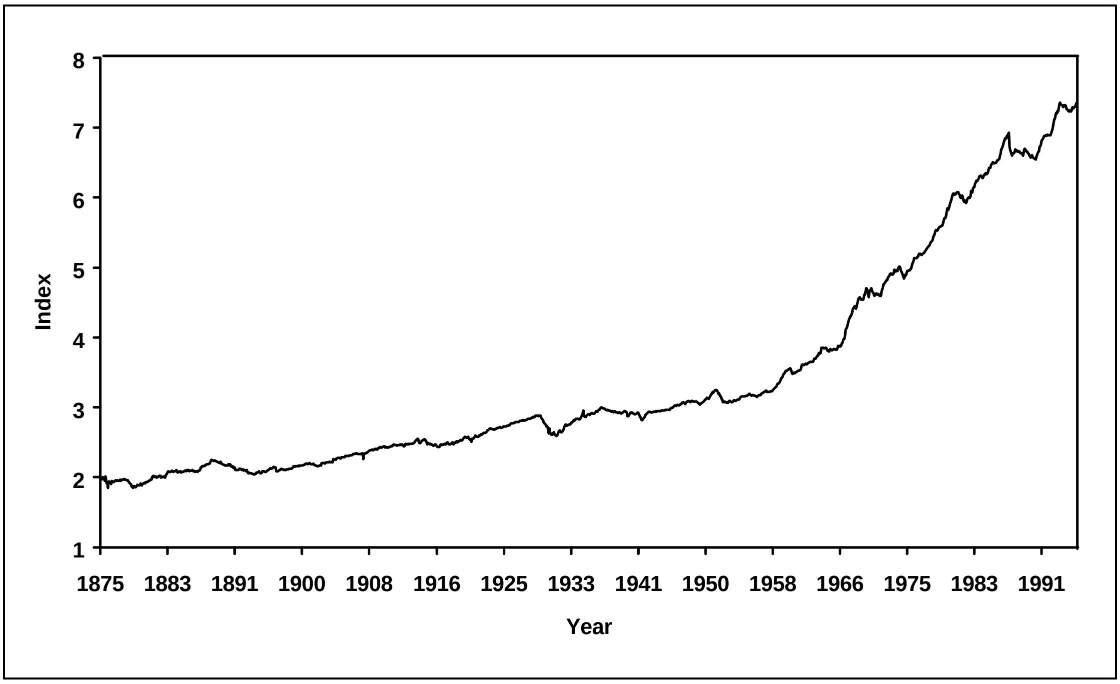


FIGURE 2 - Monthly Data

The following figure presents a plot of the log(R/S) and log(E(R/S)) values against log(n) taken from Table 1.

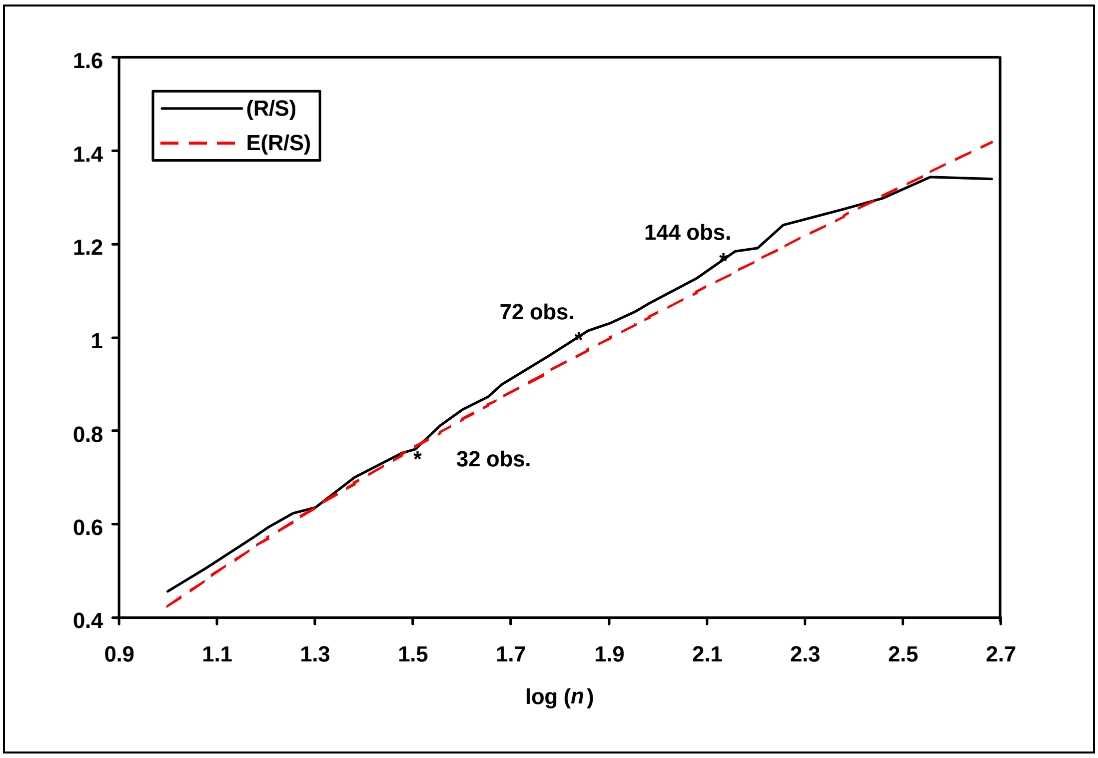


FIGURE 3 - Monthly Data

The following figure presents a plot of the V-statisic for R/S and E(R/S) against log(n) taken from Table 1.

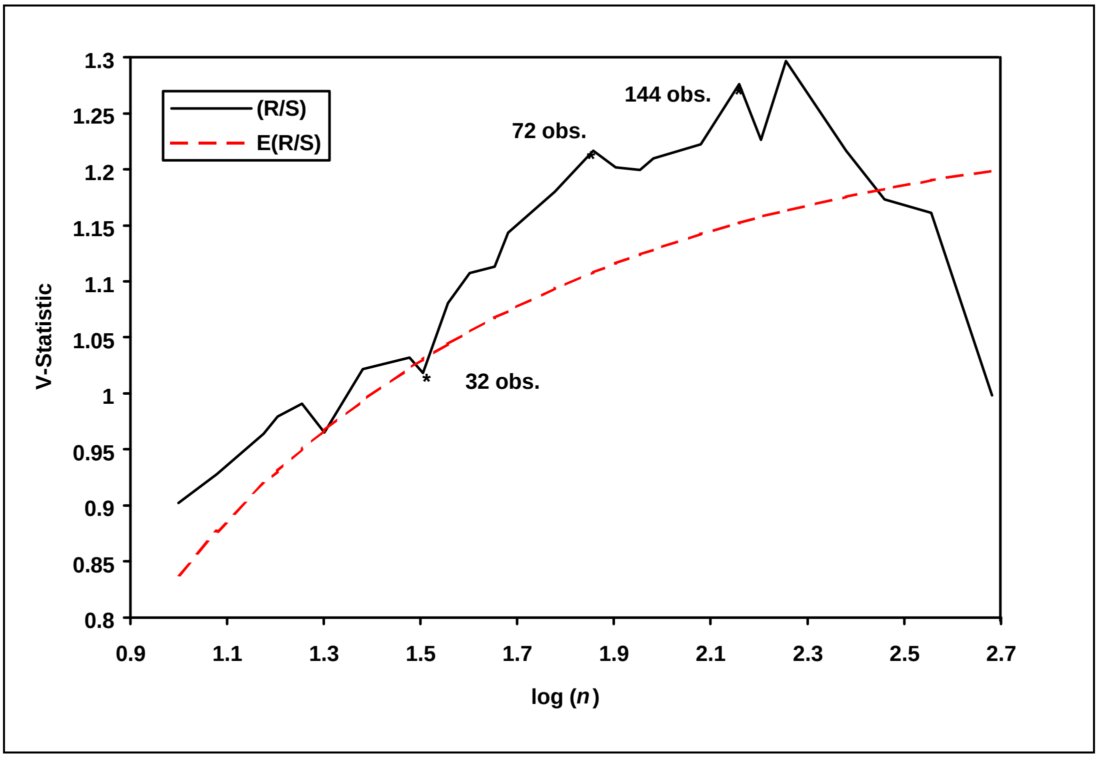


FIGURE 4 - Daily Data

The following figure presents a plot of the log(R/S) and log(E(R/S)) values against log(n) taken from Table 3.

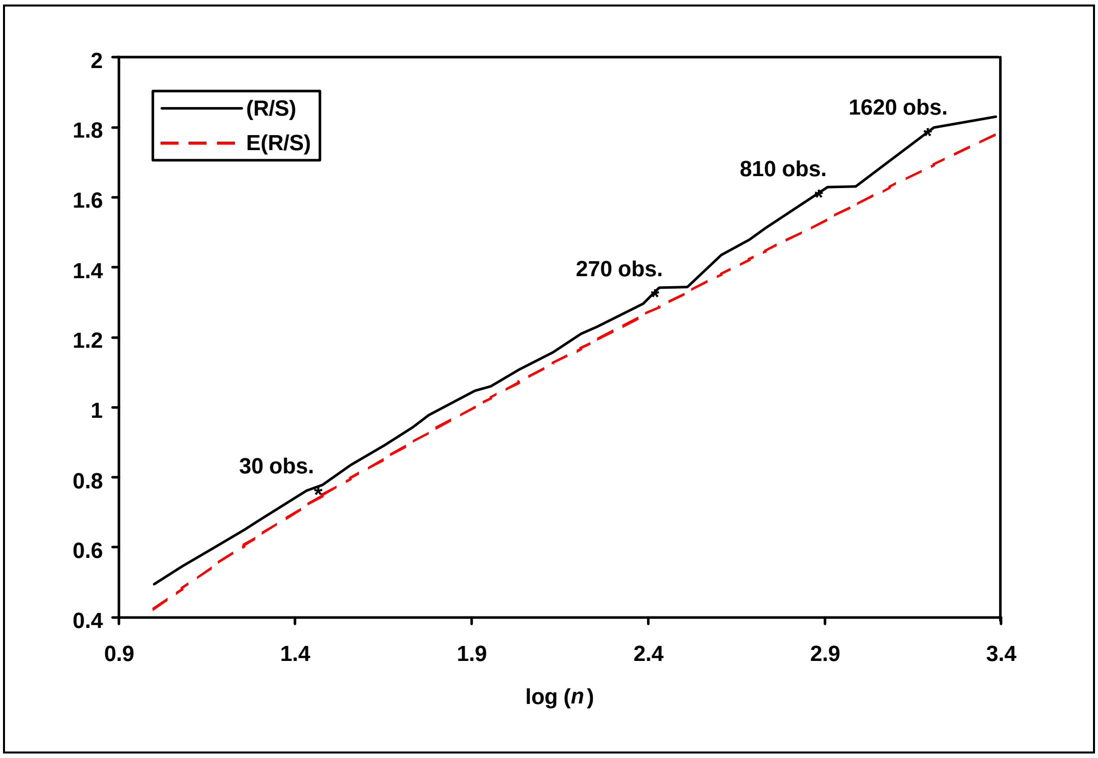
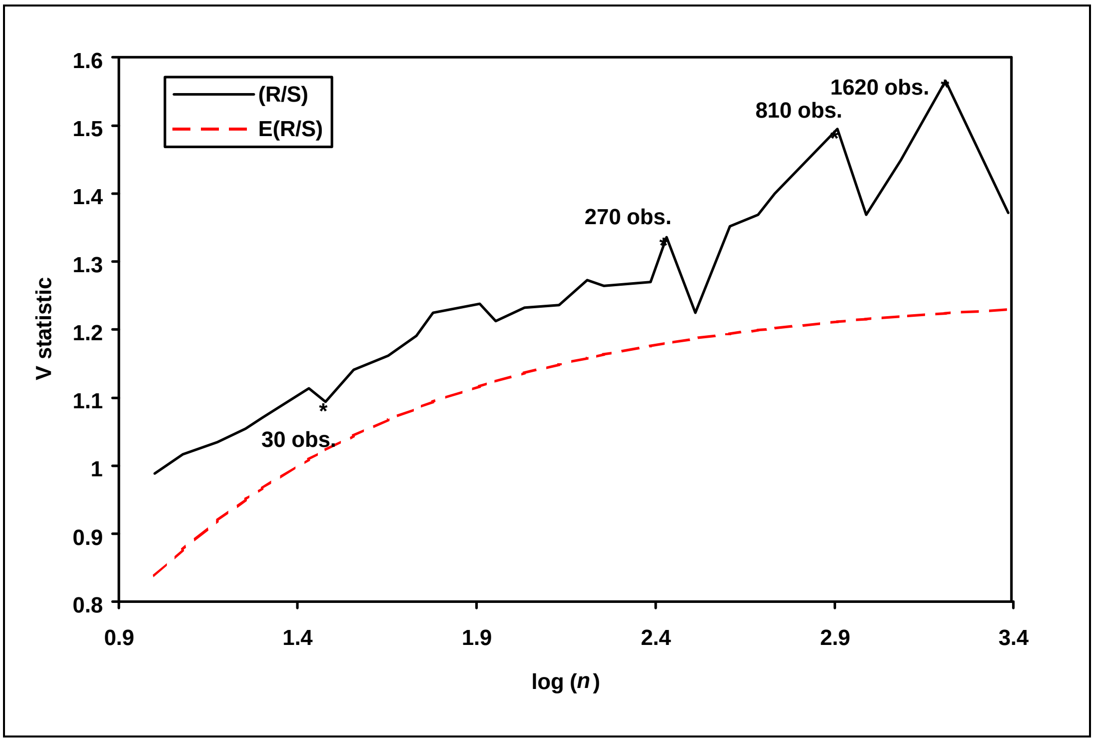


FIGURE 5 - Daily Data

The following figure presents a plot of the V-statisic for R/S and E(R/S) against log(n) taken from Table 3.



5 BIBLIOGRAPHY

Anis, A.A. and Lloyd, E.H. (1976) “The Expected Value of the Adjusted Rescaled Range of Independent Normal Summands” *Biometrika*, 63.

Batten, J. and Ellis, C. (1996) “Fractal Structures and Naïve Trading Systems: Evidence from the spot US dollar/Japanese yen” *Japan and the World Economy*, 8, pp. 411 - 421.

Brailsford, T.J. and Easton, S. (1991) “Seasonality in Australian Share Price Indices Between 1936 and 1957”*Accounting and Finance*, 31, pp. 69 - 85

Brailsford, T.J. and Faff, R.W. (1993) “Modelling Australian Stock Market Volatility” *Australian Journal of Management*, 18, 109-132.

Brock, W.A., Dechert, W.D., Sheinkman, J.A. and Le Baron, B. (1996) “A Test for Independence Based on Correlation Dimension” *Econometric Reviews*, 15.

Brock, W.A. and de Lima, P.J.F. (1996) “Nonlinear Time Series, Complexity Theory and Finance” in Maddala, G.S. and Rao, C.R. eds. *Handbook of Statistics,* Vol 14, Elsevier Science Publishing.

Cheung, Y.W. and Lai, K.S. (1993) “Do Gold Market Returns Have Long Memory?” *The Financial Review*, 28, pp. 181 - 202.

Corazza, M., Malliaris, A.G. and Nardelli, C. (1997) “Searching for Fractal Structure in Agricultural Futures Markets” *The Journal of Futures Markets*, 17, pp. 433 - 473.

Cutland, N.J., Kopp, P.E. and Willinger, W. (1993) “Stock Price Returns and the Joseph Effect: Fractional version of the Black-Scholes model” *Mathematics Research Reports*, pp. 297 - 306.

Einstein, A. (1908) “Uber die von der Molekularkinetischen Theorie der Warme Geforderte Bewegung von in Ruhenden Flussigkeiten Suspendierten Teilchen” *Annals of Physics*, 322.

Hiemstra, C. and Jones, J. (1994) “Another Look at Long Memory in Common Stock Returns” Discussion Paper 94/077, University of Strathclyde.

Huang, B.N. and Yang, C.W. (1995) “The Fractal Structure in Multinational Stock Returns” *Applied Economics Letters*, 2, pp. 67 - 71.

Hurst, H.E. (1951) “The Long Term Storage Capacity of Reservoirs” *Transactions of the American Society of Civil Engineers*, 116.

Layton, A. (1997a) “A New Approach to Dating and Predicting Australian Business Cycle Phase Changes” *Applied Economics*, 29, pp. 861 - 868.

Layton, A. (1997b) “Do Leading Indicators Really Predict Australian Business Cycle Turning Points” *Economic Record*, 73, pp. 258 - 269.

Lo, A.W. (1991) “Long-Term Memory in Stock Market Prices” *Econometrica*, 59, pp. 1279 - 1313.

Mandelbrot, B. (1972) “Statistical Methodology for Non-Periodic Cycles: from the covariance to R/S analysis” *Annals of Economic and Social Measurement*, 1.

Mandelbrot, B. (1982) “The Fractal Geometry of Nature” W.H. Freeman, New York.

Mandelbrot, B. and Wallis, J. (1969) “Robustness of the Rescaled Range R/S in the Measurement of Noncyclic Long Run Statistical Dependence” *Water Resources Research*, 5.

Peters, E.E. (1994) “Fractal Market Analysis : Applying chaos theory to investment and economics” John Wiley and Sons, New York.